REPASO 2 - Ejercicio 1

Ej.1 Calcular:

a) $\int \left(\frac{1}{x^2}.(\sqrt{x}+x) + 6x.\cos(x^2) - 5x.\cos(x)\right) dx$

$$\boxed{\int \frac{1}{x^2}(\sqrt{x}+x)\,dx} + \boxed{\int 6x\cos(x^2)\,dx} - \boxed{\int 5x\cos(x)\,dx} =$$

1 2 3

$$\alpha \neq -1 \qquad \int x^{\alpha}\,dx = \frac{x^{\alpha+1}}{\alpha+1} + C \qquad \alpha+1 \neq 0$$

1 $\int\left(\frac{1}{x^2}\cdot\sqrt{x} + \frac{1}{x^2}\cdot x\right)dx =$

$$\int (x^{-2}\cdot x^{\frac{1}{2}} + x^{-2}\cdot x^{1})\,dx = \int (x^{-2+\frac{1}{2}} + x^{-2+1})\,dx =$$

$$= \int (x^{-\frac{3}{2}} + x^{-1})\,dx = \frac{x^{(-\frac{3}{2}+1)}}{(-\frac{3}{2}+1)} + \cancel{\frac{x^{-1+1}}{-1+1}} + C$$

$$= \int \left(x^{-3/2} + \frac{1}{x}\right) dx = \frac{x^{-1/2}}{-1/2} + \boxed{\ln|x|} + C$$

$$= \boxed{(-2)\cdot x^{-1/2} + \ln|x| + C} \quad 1$$

2 $\int 6x.\cos(\overset{u}{x^2})\,dx = 6.\int \cos(x^2)\,.\,\boxed{x\,dx} =$

Sust:

$u = (x^2)$

$du = [x^2]'\,dx$

$du = 2x\,dx$

$\boxed{\frac{1}{2}du} = x\,dx$

$$= 6\int \cos(u)\,\frac{1}{2}\,du =$$

$$= \cancel{6}\cdot\frac{1}{\cancel{2}}\int \cos(u)\,du =$$

$$\boxed{\frac{1}{2}du} = x\,dx$$

$$= 3 \cdot sen(u) + C$$

$$= \boxed{3 \cdot sen(x^2) + C} \quad (2)$$

(3) $\int \overset{g}{5x} \cdot \overset{f'}{\cos(x)}\,dx =$

$$\boxed{\int f' g = f \cdot g - \int f \cdot g'} \quad \text{Partes}$$

$$= \int \overset{f'}{\cos(x)} \cdot \overset{g}{5x}\,dx = sen(x) \cdot 5x - \int \overset{f}{sen(x)} \cdot \overset{g'}{5}\,dx$$

Partes:

$f'(x) = \cos(x) \longrightarrow f(x) = sen(x)$

$g(x) = 5x \rightarrow g'(x) = 5$

$$= sen(x) . 5x - 5 . \int sen(x)\,dx$$

$$= sen(x) 5x \quad 5 . (-\cos(x)) + C$$

$$= \boxed{sen(x) 5x + 5\cos(x) + C} \quad (3)$$

$$\Rightarrow \int \left(\frac{1}{x^2} . (\sqrt{x} + x) + 6x . \cos(x^2) - 5x . \cos(x)\right) dx =$$

$$= \boxed{(-2) . x^{-1/2} + \ln|x| + 3 \cdot sen(x^2) - sen(x) 5x - 5\cos(x) + C}$$

b) $\int \left(\frac{x}{(5x^2+1)^3} + x^3 . \ln(x)\right) dx$

$$= \boxed{\int \frac{x}{(5x^2+1)^3}\,dx} + \boxed{\int x^3 . \ln(x)\,dx}$$

(1) (2)

(1) $\int \frac{x}{(5x^2+1)^3}\,dx = \int \frac{1}{(5x^2+1)^3} . \boxed{x\,dx} =$

$$u = 5x^2 + 1$$
$$du = \underbrace{(5x^2+1)'}\,dx$$
$$du = 10x\,dx$$
$$\boxed{\frac{1}{10}du} = x\,dx$$

$$= \int \frac{1}{u^3}\boxed{\frac{1}{10}du} = \frac{1}{10}\int \frac{1}{u^3}du$$

$$= \frac{1}{10}\int u^{-3}du = \frac{1}{10}\cdot\frac{u^{-3+1}}{-3+1} + C$$

$$= \frac{1}{10}\frac{u^{-2}}{(-2)} + C = \frac{1}{10}\frac{(5x^2+1)^{-2}}{(-2)} + C$$

$$= \boxed{-\frac{1}{20}\cdot(5x^2+1)^{-2} + C} \quad (1)$$

$\cancel{f'}$!!

$$\int f'\cdot g = f\cdot g - \int f\cdot g'$$ Partes

(2) $$\int \underbrace{x^3}_{f'}\cdot\underbrace{\ln(x)}_{g}\,dx =$$

$f'(x) = x^3 \rightarrow f(x) = \frac{x^4}{4}$

$g(x) = \ln(x) \rightarrow g'(x) = \frac{1}{x}$

$$= \frac{x^4}{4}\cdot\ln(x) - \int \frac{\cancel{x^4}^{\,3}}{4}\cdot\frac{1}{\cancel{x}}\,dx$$

$$= \frac{x^4}{4}\cdot\ln(x) - \int \frac{x^3}{4}\,dx$$

$$= \frac{x^4}{4}\cdot\ln(x) - \int \frac{1}{4}\cdot x^3\,dx$$

$$= \frac{x^4}{4}\ln(x) - \frac{1}{4}\cdot\int x^3dx = \frac{x^4}{4}\ln(x) - \frac{1}{4}\cdot\frac{x^4}{4} + C$$

$$= \boxed{\frac{x^4}{4}\ln(x) - \frac{1}{16}\cdot x^4 + C} \quad (2)$$

$$\Rightarrow \int\left(\frac{x}{(5x^2+1)^3}+x^3.\ln(x)\right)dx = \boxed{-\frac{1}{20}\cdot(5x^2+1)^{-2}+\frac{x^4}{4}\ln(x)-\frac{1}{16}\cdot x^4+C}$$

c) $\int \sqrt{2x-6}\,dx$

$$\int\sqrt{2x-6}\,dx = \int (2x-6)^{1/2}\,\boxed{dx} = \int u^{1/2}\cdot\boxed{\frac{1}{2}du}$$

$u = (2x-6)$

$du = 2\,dx$

$\boxed{\frac{1}{2}du = dx}$

$$= \frac{1}{2}\cdot\int u^{1/2}du = \frac{1}{2}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2}+C = \frac{1}{2}\,\frac{(2x-6)^{3/2}}{3/2}+C$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot(2x-6)^{3/2}+C =$$

$$\Rightarrow \int\sqrt{2x-6}\,dx = \boxed{\frac{1}{3}\cdot(2x-6)^{3/2}+C}$$

d) $\int(\cos(7x)+8e^{3x}+2e^{-x})dx$

$$= \int \cos(7x)\,dx + 8\cdot\boxed{\int e^{3x}dx} + 2\cdot\int e^{-x}dx$$

2

2 $\int e^{3x}dx = \cancel{e^{3x}} + c$

$= \frac{1}{3}\cdot e^{3x}+c$ ✓

$[e^{3x}]' = \cancel{e^{3x}}$

$[e^{3x}]' = e^{3x}\cdot 3$

$[\frac{1}{3}e^{3x}]' = \frac{1}{3}e^{3x}\cdot 3$ ✓

Con Sust:

$$\int e^{3x}\,dx = \int e^{u}\cdot\frac{1}{3}\,du =$$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$= \frac{1}{3}\int e^{u}\,du =$$

$$= \frac{1}{3}e^{u} + C = \boxed{\frac{1}{3}e^{3x}} + C$$

Análogamente:

$$\int \underbrace{\cos(7x)}\,dx = \frac{1}{7}\operatorname{sen}(7x) + C$$

$$\int f(x)\,dx = g(x) + C$$

tq $\quad g'(x) = f(x)$

$$\left[\frac{1}{7}\operatorname{sen}(7x)\right]' = \frac{1}{7}\left[\operatorname{sen}(7x)\right]'$$

$\int \quad '$

$$= \frac{1}{\cancel{7}}\cdot\cos(7x)\cdot\cancel{7} = \cos(7x)$$

- $$\int e^{-x}\,dx = \cancel{e^{-x}} + C = -e^{-x} + C$$

$u = -x$

$du = (-1)\,dx$

$-du = dx$

$$\Rightarrow \int (\cos(7x) + 8e^{3x} + 2e^{-x})\,dx = \frac{1}{7}\operatorname{sen}(7x) + 8\cdot\frac{1}{3}e^{3x} + 2\cdot\frac{e^{-x}}{(-1)} + C$$

$$= \boxed{\frac{1}{7}\operatorname{sen}(7x) + \frac{8}{3}e^{3x} - 2e^{-x} + C}$$