

REPASO 2 - Ejercicio 2

Ej2) Hallar una función $g(x)$ tal que $g'(x) = 5x \cdot \sqrt{2x-6}$ y $g(5) = 11$

Derivar
 $F \longrightarrow F'$

Integrar
 $G' \xrightarrow{\int} G$

Es decir:

$$G(x) = \int G'(x) dx$$

1º) Calculamos:

PARTES:

$$\int f' \cdot g = f \cdot g - \int f \cdot g'$$

$$\int \overbrace{5x}^g \cdot \overbrace{\sqrt{2x-6}}^{f'} dx = \underbrace{f(x)}_{?} \cdot \overbrace{5x}^g - \int \underbrace{5}_{g'} \cdot \underbrace{f(x)}_{?} dx$$

$$g(x) = 5x \Rightarrow g'(x) = 5$$

$$f'(x) = \sqrt{2x-6} \Rightarrow f(x) = ? \text{ Hay que Integrar } f'$$

$$\Rightarrow f(x) = \int \sqrt{2x-6} dx = \dots \text{ CÁLCULO AUXILIAR (Sustitución) } \dots = \frac{1}{3} (2x-6)^{3/2} = f(x)$$

(VER EJ. 1c)

$$\Rightarrow \int \overbrace{5x}^g \cdot \overbrace{\sqrt{2x-6}}^{f'} dx = \overbrace{\frac{1}{3} (2x-6)^{3/2}}^f \cdot \overbrace{5x}^g - \int \underbrace{5}_{g'} \cdot \underbrace{\frac{1}{3} (2x-6)^{3/2}}_f dx$$

$$= \frac{1}{3} (2x-6)^{3/2} \cdot 5x - \frac{5}{3} \cdot \int (2x-6)^{3/2} dx$$

3/2

\dots
 $\dots \left(\frac{3}{2} + 1\right)$

Sust.

$$= \frac{1}{3} (2x-6)^{3/2} \cdot 5x - \frac{5}{3} \cdot \frac{1}{2} \cdot \frac{(2x-6)^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$$

$$= \frac{1}{3} \cdot 5x \cdot (2x-6)^{3/2} - \frac{5}{3} \cdot \frac{1}{2} \cdot \frac{(2x-6)^{5/2}}{5/2} + C$$

$$\Rightarrow g(x) = \frac{5x}{3} \cdot (2x-6)^{3/2} - \frac{5}{3} \cdot \frac{1}{2} \cdot \frac{2}{5} \cdot (2x-6)^{5/2} + C$$

$$\Rightarrow g(x) = \frac{5x}{3} \cdot (2x-6)^{3/2} - \frac{1}{3} (2x-6)^{5/2} + C \quad (C \in \mathbb{R})$$

2º) Falta **hallar C** \longrightarrow usamos el dato (enunciado): **$g(5)=11$**

$$g(5) = 11$$

$$g(5) = \frac{5}{3} \cdot 5 \cdot (2 \cdot 5 - 6)^{3/2} - \frac{1}{3} (2 \cdot 5 - 6)^{5/2} + C$$

$$= \frac{25}{3} \cdot 4^{3/2} - \frac{1}{3} \cdot 4^{5/2} + C$$

$$= \frac{25}{3} \cdot 8 - \frac{1}{3} \cdot 32 + C$$

$$4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$$

$$4^{5/2} = (\sqrt{4})^5 = 2^5 = 32$$

$$\Rightarrow g(5) = \frac{200}{3} - \frac{32}{3} + C = \frac{168}{3} + C = 11$$

$$\Rightarrow C = 11 - \frac{168}{3} = -\frac{135}{3} = -45$$

$$\Rightarrow g(x) = \frac{5x}{3} \cdot (2x-6)^{3/2} - \frac{1}{3} (2x-6)^{5/2} - 45$$