INTEGRALES (Repaso)

Ej1) Calcular:

a) $\int \left(\frac{1}{x^2}.\left(\sqrt{x}+x\right)+6x.cos(x^2)-5x.cos(x)\right)dx$

b) $\int \left(\frac{x}{(5x^2+1)^3}+x^3.\ln(x)\right)dx$

c) $\int \sqrt{2x-6}\,dx$

d) $\int (\cos(7x)\ +8e^{3x}\ +2e^{-x})dx$

Ej2) Hallar una función $g(x)$ tal que $g'(x)=5x.\sqrt{2x-6}$ y $g(5)=11$

Ej3) Hallar el área de la región limitada por: los ejes coordenados; la recta $y=2$ y el gráfico de $f(x)=\sqrt{2x-6}$.

Ej4) Hallar el área de la región limitada por las curvas: $y=x-2$; $x=1$; $y=\frac{15}{x}$

REPASO 2 - Ejercicio 1

Ej.1 Calcular:

a) $\int \left(\frac{1}{x^2}.(\sqrt{x}+x)+6x.\cos(x^2)-5x.\cos(x)\right)dx$

$$\boxed{\int \frac{1}{x^2}(\sqrt{x}+x)\,dx}_{①} + \boxed{\int 6x\cos(x^2)\,dx}_{②} - \boxed{\int 5x\cos(x)\,dx}_{③} =$$

$$\boxed{\alpha \neq -1 \qquad \int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + C \qquad \alpha+1 \neq 0}$$

① $\int \left(\frac{1}{x^2}\cdot\sqrt{x} + \frac{1}{x^2}\cdot x\right)dx =$

$$\int \left(x^{-2}\cdot x^{\frac{1}{2}} + x^{-2}\cdot x^{1}\right)dx = \int \left(x^{-2+\frac{1}{2}} + x^{-2+1}\right)dx =$$

$$= \int \left(x^{-\frac{3}{2}} + x^{-1}\right)dx = \frac{x^{(-\frac{3}{2}+1)}}{(-\frac{3}{2}+1)} + \cancel{\frac{x^{-1+1}}{-1+1}} + C$$

$$= \int \left(x^{-3/2} + \frac{1}{x}\right)dx = \frac{x^{-1/2}}{-1/2} + \boxed{\ln|x|} + C$$

$$= \boxed{(-2)\cdot x^{-1/2} + \ln|x| + C} \quad ①$$

② $\int 6x.\cos(\overset{u}{x^2})\,dx = 6.\int \cos(x^2)\,.\,\boxed{x\,dx} =$

Sust:

$u = (x^2)$

$du = [x^2]'dx$

$\to du = 2x\,dx$

$\boxed{\frac{1}{2}du} = x\,dx$

$$= 6\int \cos(u)\,\frac{1}{2}du =$$

$$= \cancel{6}\cdot\frac{1}{\cancel{2}}\boxed{\int \cos(u)\,du} =$$

$$\boxed{\frac{1}{2}du} = x\,dx$$

$$= 3 \cdot \operatorname{sen}(u) + C$$

$$= \boxed{3 \cdot \operatorname{sen}(x^2) + C} \quad (2)$$

(3) $\int \overset{g}{5x} \cdot \overset{f'}{\cos(x)}\,dx =$

$$\int f' g = f \cdot g - \int f \cdot g' \quad \text{Partes}$$

$$= \int \overset{f'}{\cos(x)} \cdot \overset{g}{5x}\,dx = \operatorname{sen}(x) \cdot 5x - \int \overset{f}{\operatorname{sen}(x)} \cdot \overset{g'}{5}\,dx$$

Partes:

$f'(x) = \cos(x) \longrightarrow f(x) = \operatorname{sen}(x)$

$g(x) = 5x \longrightarrow g'(x) = 5$

$$= \operatorname{sen}(x) \cdot 5x - 5 \cdot \int \operatorname{sen}(x)\,dx$$

$$= \operatorname{sen}(x)\,5x \quad 5 \cdot (-\cos(x)) + C$$

$$= \boxed{\operatorname{sen}(x)\,5x + 5\cos(x) + C} \quad (3)$$

$$\Rightarrow \int \left(\frac{1}{x^2} \cdot (\sqrt{x} + x) + 6x.\cos(x^2) - 5x.\cos(x) \right) dx =$$

$$= \boxed{(-2) \cdot x^{-1/2} + \ln|x| + 3 \cdot \operatorname{sen}(x^2) - \operatorname{sen}(x)\,5x - 5\cos(x) + C}$$

b) $\int \left(\frac{x}{(5x^2+1)^3} + x^3.\ln(x) \right) dx$

$$= \boxed{\int \frac{x}{(5x^2+1)^3}\,dx} + \boxed{\int x^3 \cdot \ln(x)\,dx}$$

(1) (2)

(1) $\int \frac{x}{(\underbrace{5x^2+1}_{u})^3}\,dx = \int \frac{1}{(\underbrace{5x^2+1}_{u})^3} \cdot \boxed{x\,dx} =$

$$(5x^2+1) \qquad (5x^2+1)^3$$

$u = 5x^2 + 1$

$du = \underbrace{(5x^2+1)'}\, dx$

$du = 10x\, dx$

$\boxed{\frac{1}{10} du} = x\,dx$

$$= \int \frac{1}{u^3} \boxed{\frac{1}{10} du} = \frac{1}{10} \int \frac{1}{u^3} du$$

$$= \frac{1}{10} \int u^{-3} du = \frac{1}{10} \cdot \frac{u^{-3+1}}{-3+1} + C$$

$$= \frac{1}{10} \frac{u^{-2}}{(-2)} + C = \frac{1}{10} \frac{(5x^2+1)^{-2}}{(-2)} + C$$

$$= \boxed{-\frac{1}{20} \cdot (5x^2+1)^{-2} + C} \quad (1)$$

~~f'~~ !!

$$\int f' \cdot g = f \cdot g - \int f \cdot g' \quad \text{Partes}$$

(2) $$\int \underbrace{x^3}_{f'} \cdot \underbrace{\ln(x)}_{g}\, dx =$$

$f'(x) = x^3 \rightarrow f(x) = \frac{x^4}{4}$

$g(x) = \ln(x) \rightarrow g'(x) = \frac{1}{x}$

$$= \frac{x^4}{4} \cdot \ln(x) - \int \frac{\cancel{x^4}^{\,3}}{4} \cdot \frac{1}{\cancel{x}} dx$$

$$= \frac{x^4}{4} \cdot \ln(x) - \int \frac{x^3}{4} dx$$

$$= \frac{x^4}{4} \cdot \ln(x) - \int \frac{1}{4} \cdot x^3 dx$$

$$= \frac{x^4}{4} \ln(x) - \frac{1}{4} \cdot \int x^3 dx = \frac{x^4}{4} \ln(x) - \frac{1}{4} \cdot \frac{x^4}{4} + C$$

$$= \boxed{\frac{x^4}{4} \ln(x) - \frac{1}{16} \cdot x^4 + C} \quad (2)$$

$$\Rightarrow \int\left(\frac{x}{(5x^2+1)^3}+x^3.\ln(x)\right)dx = \boxed{-\frac{1}{20}\cdot(5x^2+1)^{-2}+\frac{x^4}{4}\ln(x)-\frac{1}{16}\cdot x^4+C}$$

c) $\int \sqrt{2x-6}\,dx$

$$\int\sqrt{2x-6}\,dx = \int(2x-6)^{1/2}\,\boxed{dx} = \int u^{1/2}\cdot\boxed{\tfrac{1}{2}du}$$

$u=(2x-6)$

$du = 2\,dx$

$\boxed{\frac{1}{2}du = dx}$

$$= \frac{1}{2}\cdot\int u^{1/2}du = \frac{1}{2}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{2}\cdot\frac{u^{3/2}}{3/2}+C = \frac{1}{2}\,\frac{(2x-6)^{3/2}}{3/2}+C$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\cdot(2x-6)^{3/2}+C =$$

$$\Rightarrow \int\sqrt{2x-6}\,dx = \boxed{\frac{1}{3}\cdot(2x-6)^{3/2}+C}$$

d) $\int(\cos(7x)+8e^{3x}+2e^{-x})dx$

$$= \int\cos(7x)\,dx + 8\cdot\boxed{\int e^{3x}dx} + 2\cdot\int e^{-x}dx$$

2

2 $\int e^{3x}dx = \cancel{e^{3x}} + c$

$= \frac{1}{3}\cdot e^{3x}+c$ ✓

$[e^{3x}]' = \cancel{e^{3x}}$

$[e^{3x}]' = e^{3x}\cdot 3$

$[\frac{1}{3}e^{3x}]' = \frac{1}{\cancel{3}}e^{3x}\cdot\cancel{3}$ ✓

Con Sust:

$$\int e^{3x}\,dx = \int e^{u}\cdot\tfrac{1}{3}\,du =$$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$$= \frac{1}{3}\int e^{u}\,du =$$

$$= \frac{1}{3}e^{u} + C = \boxed{\frac{1}{3}e^{3x}} + C$$

Análogamente:

$$\int \underbrace{\cos(7x)}\,dx = \frac{1}{7}\,\text{sen}(7x) + C$$

$$\int f(x)\,dx = g(x) + c$$

$$\text{tq} \quad g'(x) = f(x)$$

$$\left[\frac{1}{7}\,\text{sen}(7x)\right]' = \frac{1}{7}\left[\text{sen}(7x)\right]'$$

$\int$ $'$

$$= \frac{1}{\cancel{7}}\cdot\cos(7x)\cdot\cancel{7} = \cos(7x)$$

- $$\int e^{-x}\,dx = \cancel{e^{-x}} + C = -e^{-x} + C$$

$u = -x$

$du = (-1)\,dx$

$-du = dx$

$$\Rightarrow \int(\cos(7x) + 8e^{3x} + 2e^{-x})\,dx = \frac{1}{7}\,\text{sen}(7x) + 8\cdot\frac{1}{3}e^{3x} + 2\cdot\frac{e^{-x}}{(-1)} + C$$

$$= \boxed{\frac{1}{7}\,\text{sen}(7x) + \frac{8}{3}e^{3x} - 2e^{-x} + C}$$

REPASO 2 - Ejercicio 2

Ej2) Hallar una función $g(x)$ tal que $g'(x) = 5x.\sqrt{2x-6}$ y $g(5) = 11$

$$F \xrightarrow{\text{Derivar}} F'$$

$$G' \xrightarrow[\int]{\text{Integrar}} G$$

Es decir: $G(x) = \int G'(x)\,dx$

1º) Calculamos:

PARTES: $\int f' \cdot g = f \cdot g - \int f \cdot g'$

$$\int \overbrace{5x}^{g}\,\overbrace{\sqrt{2x-6}}^{f'}\,dx = \underbrace{f(x)}_{?} \cdot \overbrace{5x}^{g} - \int \underbrace{5}_{g'} \cdot \underbrace{f(x)}_{?}\,dx$$

$g(x) = 5x \Rightarrow g'(x) = 5$

$f'(x) = \sqrt{2x-6} \Rightarrow f(x) = ?$ Hay que Integrar f'

$\Rightarrow f(x) = \int \sqrt{2x-6}\,dx = \dots$ **CÁLCULO AUXILIAR** (Sustitución) $\dots = \frac{1}{3}(2x-6)^{3/2} = f(x)$

(VER EJ. 1c)

$$\Rightarrow \int \overbrace{5x}^{g}\,\overbrace{\sqrt{2x-6}}^{f'}\,dx = \overbrace{\frac{1}{3}(2x-6)^{3/2}}^{f} \cdot \overbrace{5x}^{g} - \int \underbrace{5}_{g'} \cdot \underbrace{\frac{1}{3}(2x-6)^{3/2}}_{f}\,dx$$

$$= \frac{1}{3}(2x-6)^{3/2}\,5x - \frac{5}{3} \cdot \boxed{\int (\underbrace{2x-6}_{u})^{3/2}\,dx}$$

Sust.

...

$3/2$ ... $(\frac{3}{2}+1)$

$$= \frac{1}{3}(2x-6)^{3/2}\cdot 5x - \frac{5}{3}\cdot \boxed{\frac{1}{2}\cdot\frac{(2x-6)^{\left(\frac{3}{2}+1\right)}}{\left(\frac{3}{2}+1\right)}} + C$$

$$= \frac{1}{3}\cdot 5x\cdot(2x-6)^{3/2} - \frac{5}{3}\cdot\frac{1}{2}\cdot\frac{(2x-6)^{5/2}}{5/2} + C$$

$$\Rightarrow g(x) = \frac{5x}{3}\cdot(2x-6)^{3/2} - \frac{\cancel{5}}{3}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{5}}\cdot(2x-6)^{5/2} + C$$

$$\Rightarrow \boxed{g(x) = \frac{5x}{3}\cdot(2x-6)^{3/2} - \frac{1}{3}(2x-6)^{5/2} + C} \quad (C\in\mathbb{R})$$

¿C?

2º) Falta **hallar C** ⟶ usamos el dato (enunciado): **g(5)=11**

$$g(5) = 11$$

$$g(5) = \frac{5}{3}\cdot 5\cdot(2\cdot 5-6)^{3/2} - \frac{1}{3}(2\cdot 5-6)^{5/2} + C$$

$$= \frac{25}{3}\cdot 4^{3/2} - \frac{1}{3}\cdot 4^{5/2} + C$$

$$= \frac{25}{3}\cdot 8 - \frac{1}{3}\cdot 32 + C$$

$$4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$$

$$4^{5/2} = \left(\sqrt{4}\right)^5 = 2^5 = 32$$

$$\Rightarrow g(5) = \frac{200}{3} - \frac{32}{3} + C = \frac{168}{3} + C = 11$$

$$\Rightarrow C = 11 - \frac{168}{3} = -\frac{135}{3} = -45$$

$$\Rightarrow \boxed{g(x) = \frac{5}{3}x\cdot(2x-6)^{3/2} - \frac{1}{3}(2x-6)^{5/2} - 45}$$

REPASO 2 - Ejercicio 3

Ej3) Hallar el área de la región limitada por: los ejes coordenados; la recta $y = 2$ y el gráfico de $f(x) = \sqrt{2x-6}$.

- Graficamos:

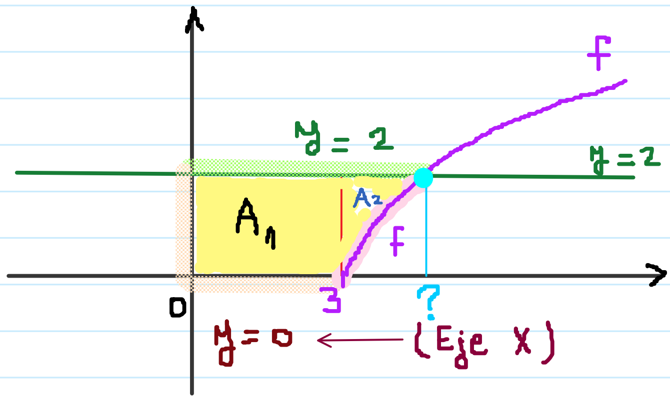
- $y = 2$ ⟵ (Recta Horizontal)
- $f(x) = \sqrt{2x-6}$ (Raíz cuadrada)

Dom f: $2x - 6 \geq 0$

$2x \geq 6$

$x \geq 3$

$\Rightarrow$ Dom f $= [3, +\infty)$



$$A_1 = \int_0^3 (\overset{\text{Techo}}{2} - \overset{\text{Piso}}{0})\, dx \quad ; \quad A_2 = \int_3^{?} (\overset{\text{Techo}}{2} - \overset{\text{Piso}}{\sqrt{2x-6}})\, dx$$

- Para terminar de plantear el Área 2, necesitamos hallar la intersección entre y=2 y el gráfico de f:

$2 = \sqrt{2x-6}$

$4 = 2x - 6$

$4 + 6 = 2x$

$10 = 2x$

$5 = x$

$$\Rightarrow A_2 = \int_3^5 (\overset{T}{2} - \overset{P}{\sqrt{2x-6}})\, dx$$

- Calculamos las Integrales:

Regla de Barrow: $$\int_a^b f(x)\,dx = G(x)\Big|_a^b = G(b) - G(a) \qquad \text{donde } G'(x) = f(x)$$

- $$A_1 = \int_0^3 (2-0)\,dx = \int_0^3 2\,dx = \overset{G(x)}{\boxed{2x}}\Big|_0^3 = 2.3 - 2.0 = 6 = A_1$$

- $$A_2 = \int_3^5 \left(2 - \sqrt{2x-6}\right) dx$$

- 1º) Calculamos la **integral <u>indefinida</u>**:

$$\int \left(2-\sqrt{2x-6}\right)dx = \int 2\,dx - \boxed{\int \sqrt{2x-6}\,dx} =$$

$\shortparallel \longleftarrow$ VER Ej 1 c)

$$= \boxed{2x - \boxed{\frac{1}{3}(2x-6)^{3/2}}} + C$$

G(x)

- 2º) Calculamos la **integral <u>definida</u>** (con la Regla de Barrow):

$$\Rightarrow A_2 = \int_3^5 \left(2-\sqrt{2x-6}\right)dx = \left(\boxed{2x - \frac{1}{3}(2x-6)^{3/2}}\right)\Bigg|_3^5 =$$

$$= \left(2.5 - \frac{1}{3}(\underbrace{2.5-6}_{4})^{3/2}\right) - \left(2.3 - \frac{1}{3}(\underbrace{2.3-6}_{=0})^{3/2}\right) =$$

$$= \left(10 - \frac{1}{3}\cdot 4^{3/2}\right) - \left(6 - \underbrace{\frac{1}{3}\cdot 0^{3/2}}_{=0}\right) =$$

$$= \left(10 - \frac{1}{3}\cdot 8\right) - (6 - 0) = 10 - \frac{8}{3} - 6 = \frac{4}{3} = A_2$$

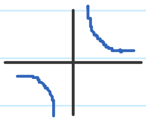
$$\Rightarrow \boxed{A_T = A_1 + A_2} = 6 + \frac{4}{3} = \boxed{\frac{22}{3} = A} \longleftarrow \text{ÁREA TOTAL}$$

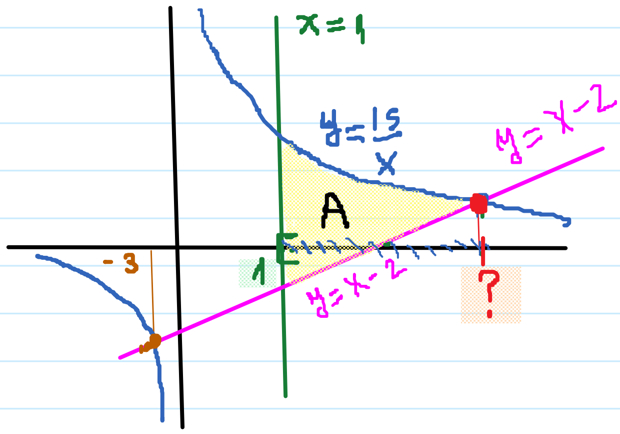
REPASO 2 - Ejercicio 4

Ej4) Hallar el área de la región limitada por las curvas: $y = x - 2$; $x = 1$; $y = \frac{15}{x}$

- Graficamos:

- $x = 1$ ⟵ **Recta Vertical**
- $y = x - 2$ ⟵ Recta (creciente)
- $y = \dfrac{15}{x}$ ⟵ Función Homográfica





$$\Rightarrow A = \int_1^{?} \Big(\overbrace{\frac{15}{x}}^{\text{Techo}} - \overbrace{(x-2)}^{\text{Piso}} \Big)\, dx$$

- Calculamos la intersección entre $y = \dfrac{15}{x}$ e $y = x - 2$:

$$\frac{15}{x} = x - 2$$

$$15 = (x-2) \cdot x$$

$$15 = x^2 - 2x$$

$$0 = x^2 - 2x - 15 \quad \begin{matrix} \nearrow x = 5 \\ \searrow \cancel{x = -3} \end{matrix}$$

$$\Rightarrow A = \int_1^{5} \Big(\overbrace{\frac{15}{x}}^{\text{Techo}} - \overbrace{(x-2)}^{\text{Piso}} \Big)\, dx$$

- Calculamos la Integral:

1º) $\int \left(\frac{15}{x} - (x-2)\right) dx =$

$= \int \left(15 \cdot \frac{1}{x} - x + 2\right) dx =$

$= 15 \cdot \int \frac{1}{x}\, dx - \int x^1 dx + \int 2\, dx =$

$= \boxed{15 \cdot \ln|x| - \frac{x^2}{2} + 2x} + C$

G(x)

Regla de Barrow: $\int_a^b f(x)\,dx = \boxed{G(x)}\Big|_a^b = G(b) - G(a)$ donde $G'(x) = f(x)$

2º) $\int_1^5 \left(\frac{15}{x} - (x-2)\, dx\right) = \boxed{\left(15 \ln|x| - \frac{x^2}{2} + 2x\right)}\Bigg|_1^5 =$

$= \left(15 \ln(5) - \frac{5^2}{2} + 2 \cdot 5\right) - \left(15 \cdot \ln(1) - \frac{1^2}{2} + 2 \cdot 1\right)$

$= \left(15 \ln(5) - \frac{25}{2} + 10\right) - \left(15 \cancel{\ln(1)} - \frac{1}{2} + 2\right) =$

$= 15 \ln(5) - \frac{25}{2} + 10 - 0 + \frac{1}{2} - 2 =$

$= 15 \ln(5) - 12 + 8 = \boxed{15 \ln(5) - 4 \cong 20,14 = A}$